DPMC: Weighted Model Counting by Dynamic Programming on Project-Join Trees [1, CP 2020]

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Unifying dynamic-programming framework for exact literal-weighted model counting

- Faster than `cachet`, `miniC2D`, `c2d`, `d4` on 584 of 1976 benchmarks (30%)

Applications

Problems solved via reduction to model counting [2]:

- Probability of disease given symptom (e.g., COVID | fever) [3, 4]
- Reliability of electricity grid (e.g., power outage in Texas during winter storm) [5]

Propositional Model Counting

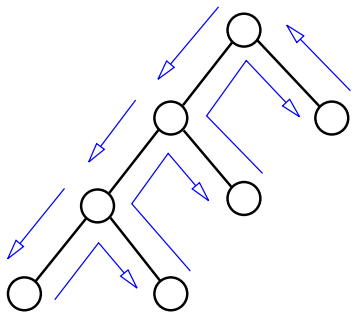
Conjunctive normal form (CNF) formula: $\varphi = x_1 \wedge (x_1 \vee \neg x_2) \wedge (\neg x_2 \vee \neg x_3)$

x_1	x_2	x_3	$\varphi(x_1, x_2, x_3)$	Model?
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	1	Yes
1	0	1	1	Yes
1	1	0	1	Yes
1	1	1	0	

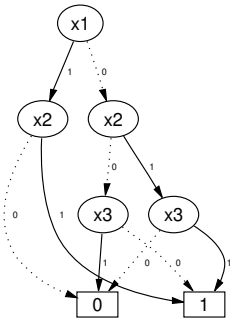
Model count: $\#\varphi = 3$ (if each literal weight is 1.0)

Related Work: Three Approaches to Model Counting

Search: explore solution space with backtracking [6] (figure from [7])



Knowledge compilation: use tractable circuit [8–10] (binary decision diagram [11], figure from [12])



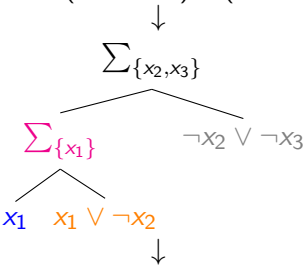
Dynamic programming: solve overlapping subproblems (e.g., `ADDMC` [13], `TensorOrder` [14])

- Contribution: unifying framework `DPMC` (dynamic-programming model counter) [1]

Model-Counting Framework: Planning Phase and Execution Phase

CNF formula $\varphi = x_1 \wedge (x_1 \vee \neg x_2) \wedge (\neg x_2 \vee \neg x_3)$

Planning phase



Project-join tree

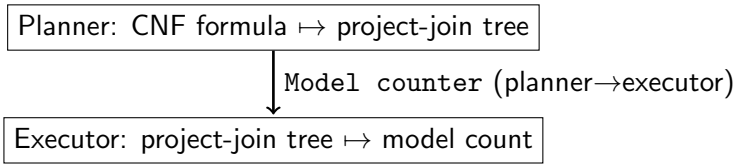
Execution phase

$$f(x_1, x_2) = x_1 \cdot (x_1 \vee \neg x_2)$$
$$g(x_2) = \sum_{x_1} f(x_1, x_2)$$
$$= f(0, x_2) + f(1, x_2)$$
$$r(x_2, x_3) = g(x_2) \cdot (\neg x_2 \vee \neg x_3)$$
$$\#\varphi = \sum_{x_2, x_3} r(x_2, x_3)$$

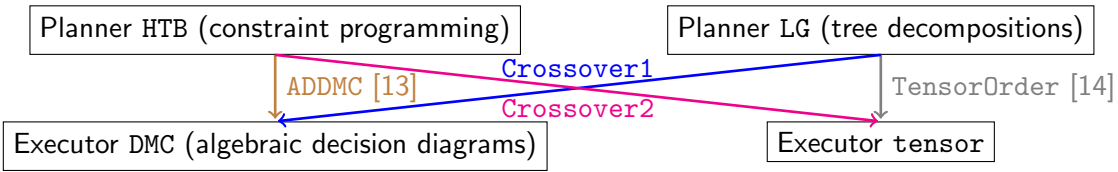
Model count $\#\varphi = 3$

Model-Counting Framework and Implementation

Framework:



Implementation:



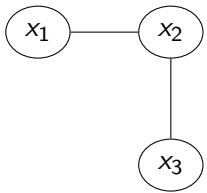
Performance:

Crossover1 > ADDMC > TensorOrder > Crossover2

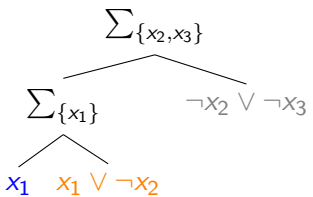
Planner HTB (Heuristic Tree Builder)

CNF formula: $x_1 \wedge (x_1 \vee \neg x_2) \wedge (\neg x_2 \vee \neg x_3)$

Primal graph



Project-join tree (**one-shot**)



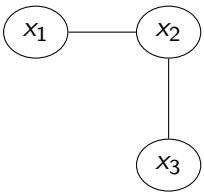
Planner HTB constructs project-join tree with constraint-programming heuristics

- Variable ordering: maximal-cardinality search [15], minimal fill-in [16]
- Clause ordering: bucket elimination [17], Bouquet's Method [18]

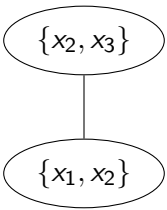
Planner LG (Line Graph)

CNF formula: $x_1 \wedge (x_1 \vee \neg x_2) \wedge (\neg x_2 \vee \neg x_3)$

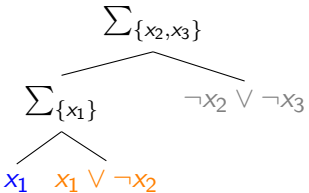
Primal graph



Tree decompositions



Project-join trees (**anytime**)



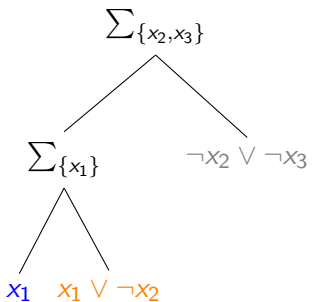
Planner LG constructs project-join trees with tree decompositions [19]

- Winning tools from heuristic-treewidth track of PACE Challenge 2017 [20]: `Tamaki` [21], `FlowCutter` [22], `htd` [23]

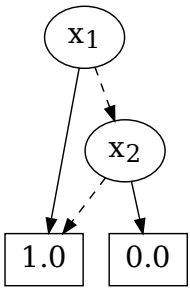
Executor DMC (Diagram Model Counter)

Node in project-join tree represents pseudo-Boolean function

Project-join tree



Algebraic decision diagram (ADD)



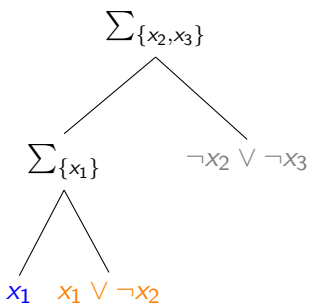
Executor DMC valuates project-join tree using **sparse** ADDs [24]

- ADD package CUDD [25]

Executor `tensor`

Node in project-join tree represents pseudo-Boolean function

Project-join tree



Tensor (multi-dimensional array)

- 0-dimension (scalar)
- 1-dimension (list)
- 2-dimension (matrix)
- ...

Executor `tensor` valuates project-join tree using **dense** tensors

- Tensor package `NumPy` [26]

Empirical Evaluation: 1976 Benchmarks (No Preprocessing)

Bayesian inference: 1080 CNF formulas [4]

- Deterministic Quick Medical Reference
- Grid Networks
- Plan Recognition

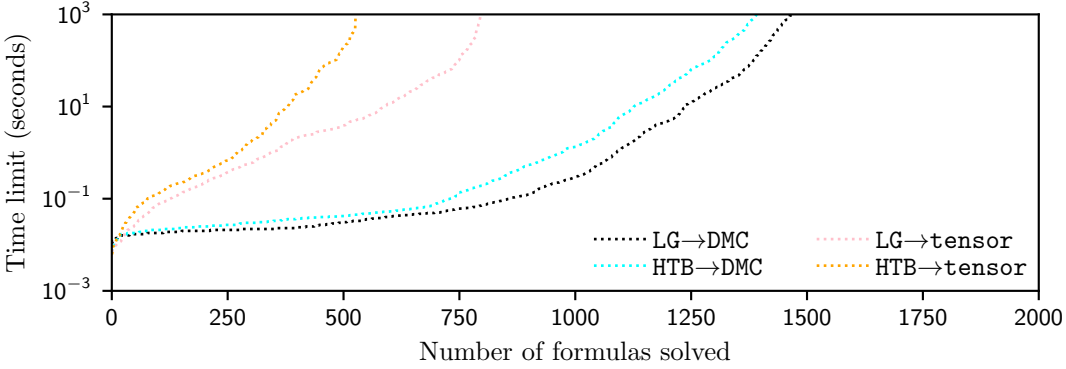
Other domains: 896 CNF formulas [27–30]

- Planning
- Bounded Model Checking
- Circuit
- Configuration
- Quantitative Information Flow
- Scheduling

Linux cluster at Rice University:

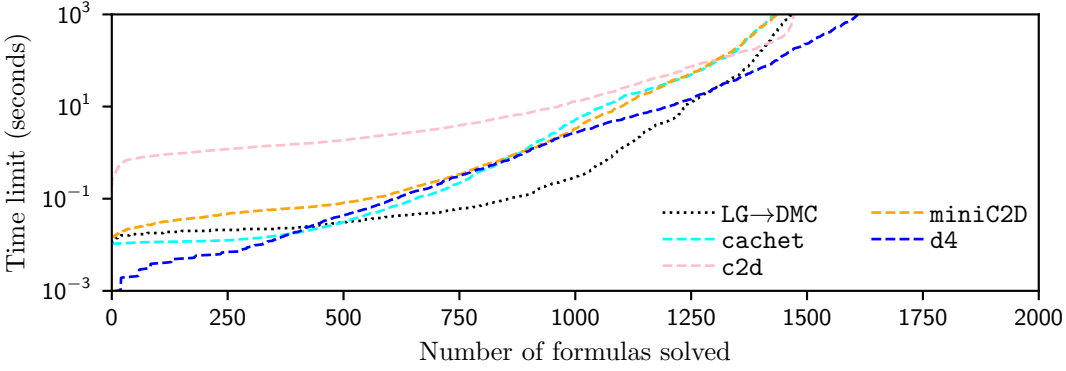
- CPU: 2.60GHz Xeon E5-2650 v2 (single-core solvers)
- RAM: 30GB

Empirical Evaluation: Planners and Executors



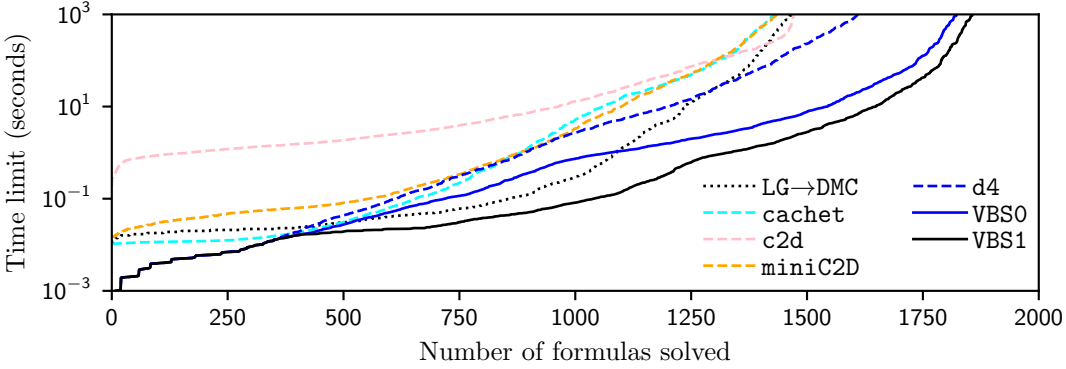
- `LG` (*anytime* tree decompositions) outperforms `HTB` (*one-shot* constraint programming)
- `DMC` (*sparse* ADDs) outperforms `tensor` (*dense* tensors)

Empirical Evaluation: Model Counters



- Exact weighted model counters: `cachet` [6], `c2d` [8], `miniC2D` [9], `d4` [10]
- `LG→DMC` fastest on 471 formulas (24%); `DPMC` (all combinations) fastest on 584 (30%)

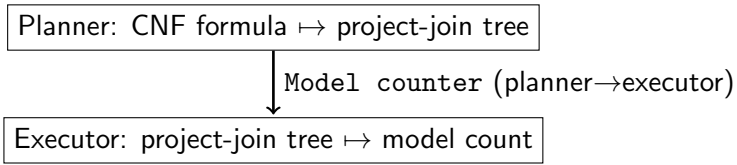
Empirical Evaluation: Virtual Best Solvers



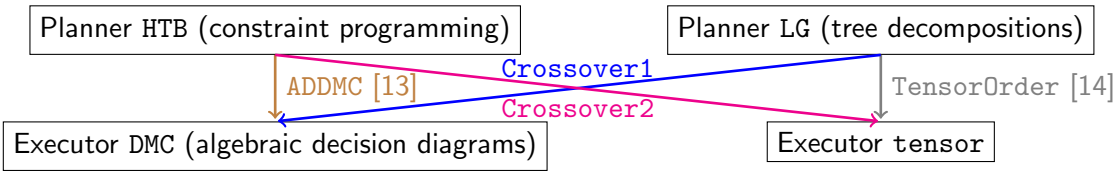
- Virtual best solver (VBS): simulation of running actual solvers in parallel
- `VBS1` (with `LG→DMC`) is faster than `VBS0` (without `LG→DMC`)

Summary: Model Counting by Dynamic Programming

Framework:



Implementation:



Model Counting Competition 2021:

- Unprojected tracks: DPMC [1, CP 2020] (https://github.com/vardigroup/DPMC)
- Projected track: ProCount [31, SAT 2021] (talk at 3:30 pm CEST on Thursday)

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