Stochastic Constraint Optimisation with Applications in Network Analysis

Extended abstract of work presented at IJCAI 2019 and DSO 2019, augmented with new work in progress.

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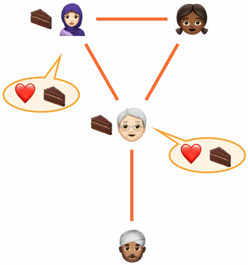
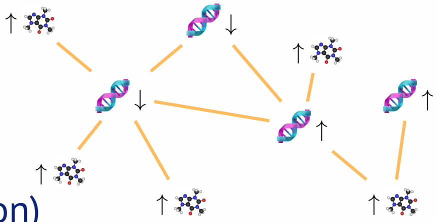
Spread-of-influence

[Domingos & Richardson 2001, Kempe *et al.*, 2003]



Signalling Regulatory Pathways

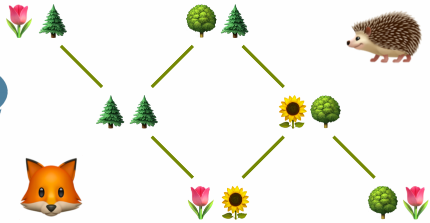
[Ourfali *et al.*, 2007]



Stochastic **C**onstraint (optimisation) **P**roblems

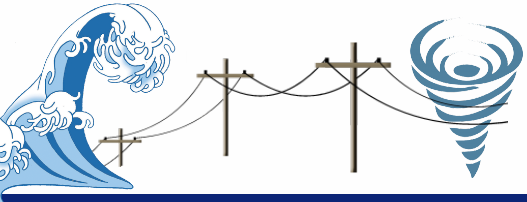
Landscape Connectivity

[Xue *et al.*, 2017]

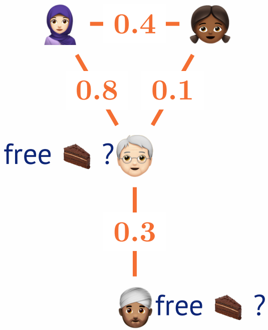


Powergrid Reliability

[Dueñas-Osorio *et al.*, 2017]



Example: Spread-of-influence problem I



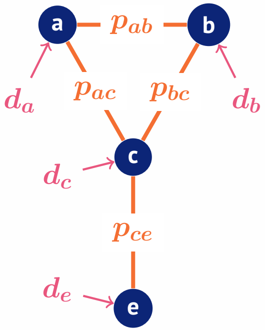
Properties

- **Probabilistic** influence;
- limited **budget** of free samples 🍰🍰;
- **maximise** expected # people buying your Sachertorte.

[Domingos & Richardson, 2001]

[Kempe *et al.*, 2003]

Example: Spread-of-influence problem II



$$P(t_{xy} = 1) = p_{xy}$$
$$P(t_{xy} = 0) = (1 - p_{xy})$$
$$d_i \in \{0, 1\}$$

Person e buys cake:

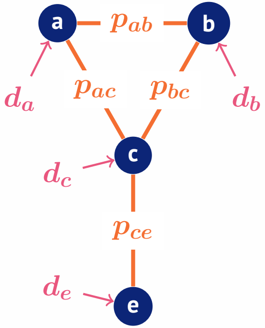
$$\begin{aligned}\phi_e = d_e \vee (d_c \wedge t_{ce}) \vee \\ (d_a \wedge t_{ac} \wedge t_{ce}) \vee \\ (d_b \wedge t_{bc} \wedge t_{ce}) \vee \\ (d_a \wedge t_{ab} \wedge t_{bc} \wedge t_{ce}) \vee \\ (d_b \wedge t_{ab} \wedge t_{ac} \wedge t_{ce})\end{aligned}$$

Find **strategy σ**:

$$\arg\max_{\sigma} \sum_{i \in \{a,b,c,e\}} P(\phi_i \mid \sigma)$$

subject to: $\sum_{i \in \{a,b,c,e\}} d_i \leq k$

Example: Spread-of-influence problem III



$$P(t_{xy} = 1) = p_{xy}$$
$$P(t_{xy} = 0) = (1 - p_{xy})$$
$$d_i \in \{0, 1\}$$

Person e buys cake:

$$\begin{aligned}\phi_e = d_e \vee (d_c \wedge t_{ce}) \vee \\ (d_a \wedge t_{ac} \wedge t_{ce}) \vee \\ (d_b \wedge t_{bc} \wedge t_{ce}) \vee \\ (d_a \wedge t_{ab} \wedge t_{bc} \wedge t_{ce}) \vee \\ (d_b \wedge t_{ab} \wedge t_{ac} \wedge t_{ce})\end{aligned}$$

repeatedly solve:

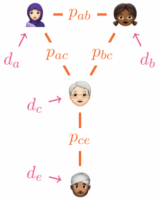
$$\boxed{\sum_{i \in \{a,b,c,e\}} P(\phi_i \mid \sigma) > \theta}$$

subject to: $\displaystyle\sum_{i \in \{a,b,c,e\}} d_i \leq k$

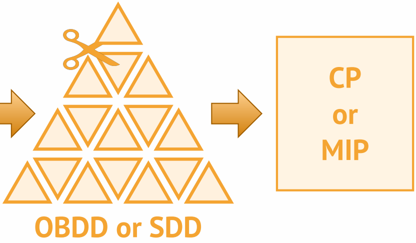
Relation to other problems

- Stochastic satisfiability (SSAT) [Papadimitriou, 1985]
- One-stage stochastic constraint satisfaction [Walsh, 2002]
- Maximum expected utility (MEU) [Dechter, 1998]
- Maximum a-posteriori (MAP) [Riedel, 2008]
- (functional) E-MAJSAT [Littman *et al.*, 1998; Pipatsrisawat & Darwiche, 2009]
- Maximum Model Counting [Fremont *et al.*, 2017]

Decomposition method



$$\begin{aligned}\phi_e = {} & d_e \vee (d_c \wedge t_{ce}) \vee \\ & (d_a \wedge t_{ac} \wedge t_{ce}) \vee \\ & (d_b \wedge t_{bc} \wedge t_{ce}) \vee \\ & (d_a \wedge t_{ab} \wedge t_{bc} \wedge t_{ce}) \vee \\ & (d_b \wedge t_{ab} \wedge t_{ac} \wedge t_{ce})\end{aligned}$$



Applicable to *any* probability distribution.
Straightforwardly implemented.

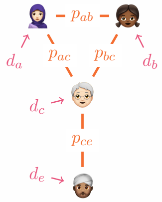
[Latour *et al.*, 2017]

Observation 1: decomposition method does **not guarantee** Generalised Arc Consistency (**GAC**) $\longrightarrow$ **inefficient**;

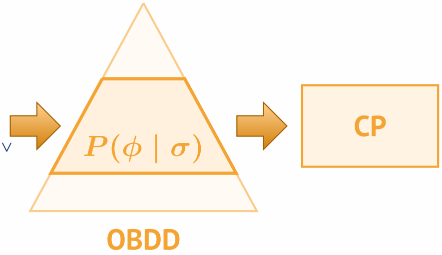
Observation 2: probability distribution is **monotonic**;

[Latour *et al.*, 2019]

Global propagation method



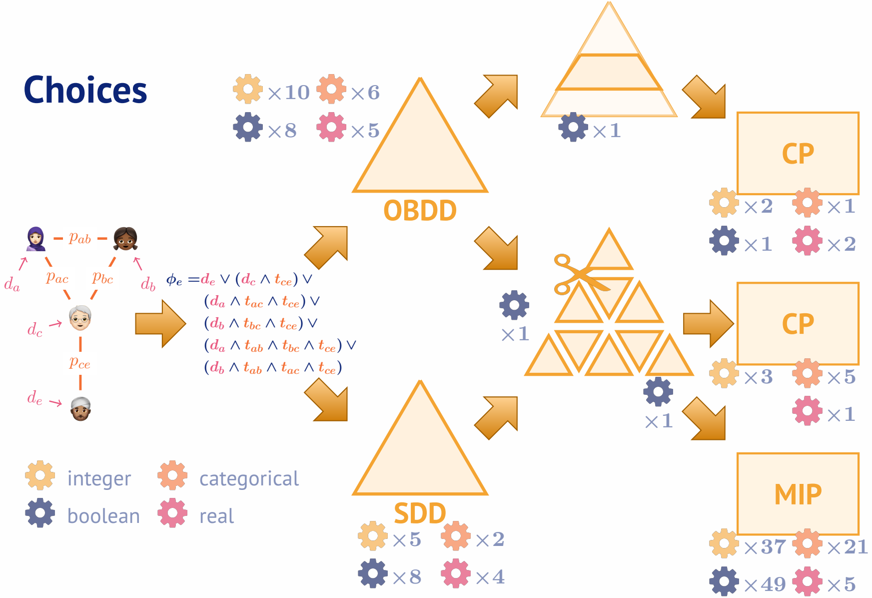
$$\begin{aligned}\phi_e = \; & d_e \vee (d_c \wedge t_{ce}) \vee \\ & (d_a \wedge t_{ac} \wedge t_{ce}) \vee \\ & (d_b \wedge t_{bc} \wedge t_{ce}) \vee \\ & (d_a \wedge t_{ab} \wedge t_{bc} \wedge t_{ce}) \vee \\ & (d_b \wedge t_{ab} \wedge t_{ac} \wedge t_{ce})\end{aligned}$$



Only for *monotonic* probability distributions.
Uses *derivatives* of free decision variables [Darwiche, 2003].
Two versions: *full-sweep* and *partial-sweep*.
Each iteration has linear time complexity.

[Latour *et al.*, 2019]

Choices



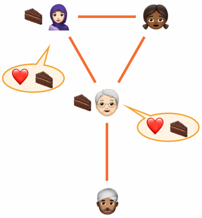
Programming by Optimisation [Hoos, 2012]
Automated Algorithm Configuration [Hoos, 2012]

[Fokkinga *et al.*, 2019]

Which optimised method is fastest?

Benchmark sets

Facebook [Viswanath *et al.*, 2009]
Spread-of-influence



High-voltage [Wiegmans, 2016]
Powergrid reliability

	# queries	# random	# decision
facebook	**15–30**	**16–107**	**15–30**
high-voltage	**6–39**	**30–300**	**15–150**

Configuration experiment

	# train	# test
facebook	**412**	**411**
high-voltage	**51**	**50**

SMAC [Hutter *et al.*, 2011]

PAR10 [CPU s] on test set (cutoff is **600** CPU s):

	CP-decomp. Gecode	MIP-decomp. Gurobi	global SCMD OscaR
facebook			
default	**4 270 (289)**	**1 664 (108)**	**782 (51)**
optimised	**2 615 (174)**	**627 (41)**	**682 (44)**
high-voltage			
default	**4 351 (36)**	**3 989 (33)**	**2 782 (23)**
optimised	**4 452 (37)**	**3 031 (25)**	**2 669 (22)**

(XXX) indicates number of unsolved instances.

How do our results generalise to harder problems?

The hardest problems

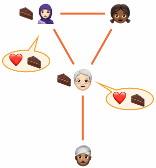
	# unsolved	# instances
facebook	**62**	**558**
high-voltage	**39**	**351**

PAR10 [CPU s] on unsolved instances (cutoff is **3600** CPU s):

	CP-decomp. Gecode	MIP-decomp. Gurobi	global SCMD OscaR
facebook			
default	**35 398 (548)**	**28 780 (441)**	**11 330 (168)**
optimised	**32 607 (504)**	**18 528 (278)**	**10 716 (158)**
high-voltage			
default	**34 325 (334)**	**33 523 (326)**	**29 300 (285)**
optimised	**32 597 (317)**	**31 302 (304)**	**29 186 (284)**

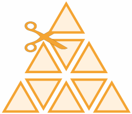
(XXX) indicates number of unsolved instances.

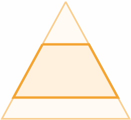
Main contribution



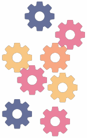
A study of solving methods for stochastic constraint (optimisation) problems, with:

- Weighted model counting for probabilistic inference.
- OBDDs or SDDs for WMC.
- Two constraint solving methods:
  - decomposition, or
  - global constraint solving.





Global constraint scales better with problem size than decomposition, but was/is less easy to implement.



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more info: A.L.D. Latour, B. Babaki, S. Nijssen, *Stochastic constraint propagation for mining probabilistic networks.* IJCAI 2019, and
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code & more **results**: `github.com/latower/SCMD`, `ada.liacs.nl/projects/scop-solver`

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