Logical Entailment for Projected Model Counting

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International Workshop on Model Counting (MCW 2020)

9 July 2020

Algorithm

Input: formula $F(X,Y)$ over variables $X \cup Y$ such that $X \cap Y = \emptyset$, trail I, decision level function δ

Output: M number of models of F projected onto X

Count(F)

$I := \varepsilon$; $\delta := \infty$; $M := 0$
forever do
    $C :=$ PropagateUnits(F, I, δ)
    if $C \neq 0$ **then**
        $c := \delta(C)$
        if $c = 0$ **then return** M
        AnalyzeConflict(F, I, C, c)
    else if all variables in $X \cup Y$ are assigned **then**
        if $V(\text{decs}(I)) \cap X = \emptyset$ **then return** $M + 2^{|X-I|}$
        $M := M + 2^{|X-I|}$
        $b := \delta(\text{decs}(\pi(I, X)))$
        Backtrack(I, $b - 1$)
    else Decide(I, δ)

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    else if Entails(I, F) **then**
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Logical Entailment Test under Projection

Given: $F(X, Y)$ formula over set of relevant variables X and set of irrelevant variables Y

I trail over variables in $X \cup Y$

Entailment under projection onto X: $\forall X \exists Y \, [F|_I]$

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Example: $F(X, Y) = x_1(x_2 \leftrightarrow y_2)$ $X = \{x_1, x_2\}$ $Y = \{y_2\}$

$F|_{x_1} = (x_2 \leftrightarrow y_2)$

$F|_{x_1 x_2} = (1 \leftrightarrow y_2)$ and $F|_{x_1 x_2 y_2} = 1$

$F|_{x_1 \overline{x_2}} = (0 \leftrightarrow y_2)$ and $F|_{x_1 \overline{x_2}\, \overline{y_2}} = 1$

$\implies$ $x_1 \models F$

Unit Propagation

Idea: Assign the propagated unit literal the decision level of its reason clause

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Unit: (F, I, M, δ) if $F|_I \neq 0$

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Unit: $(F, I, M, \delta) \rightsquigarrow_{\mathsf{Unit}} (F, \quad M, \quad)$ if $F|_I \neq 0$

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Idea: Assign the propagated unit literal the decision level of its reason clause

Unit: $(F, I, M, \delta) \rightsquigarrow_{\text{Unit}} (F, I\ell, M, \qquad)$ if $F\|_I \neq 0$ and exists $C \in F$ with $\{\ell\} = C\|_I$

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Backtracking upon Model Found

Given: Formula $F(X, Y)$ over relevant variables X and irrelevant variables Y

Idea: Flip the last relevant decision literal

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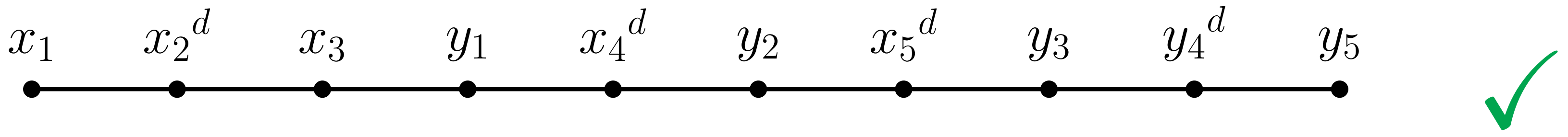

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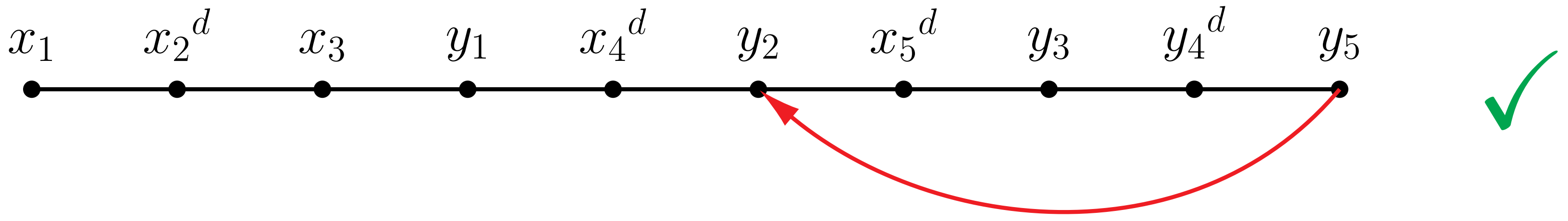

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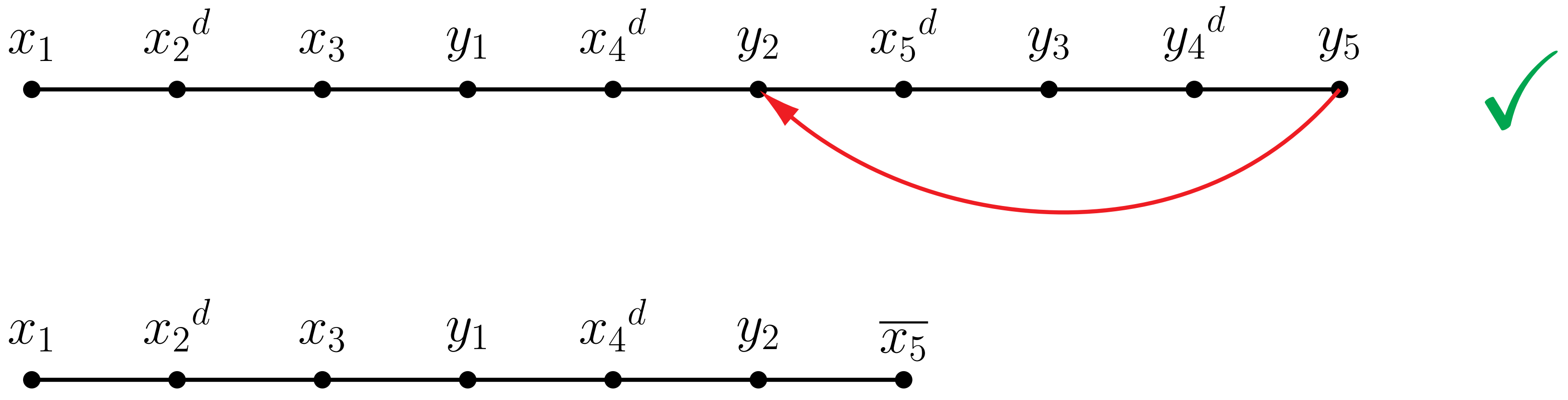

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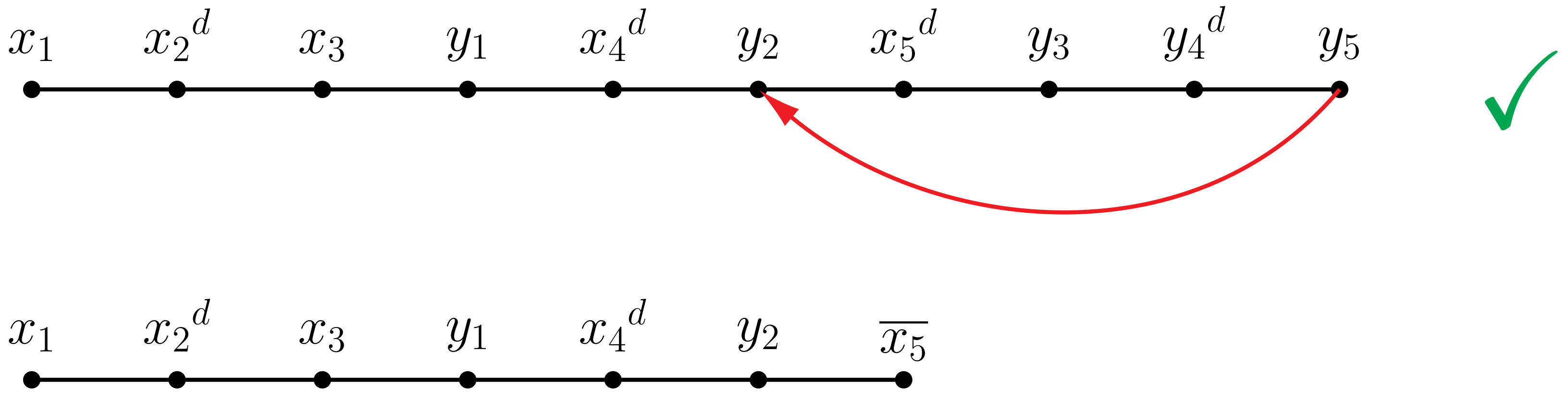

BackTrue:

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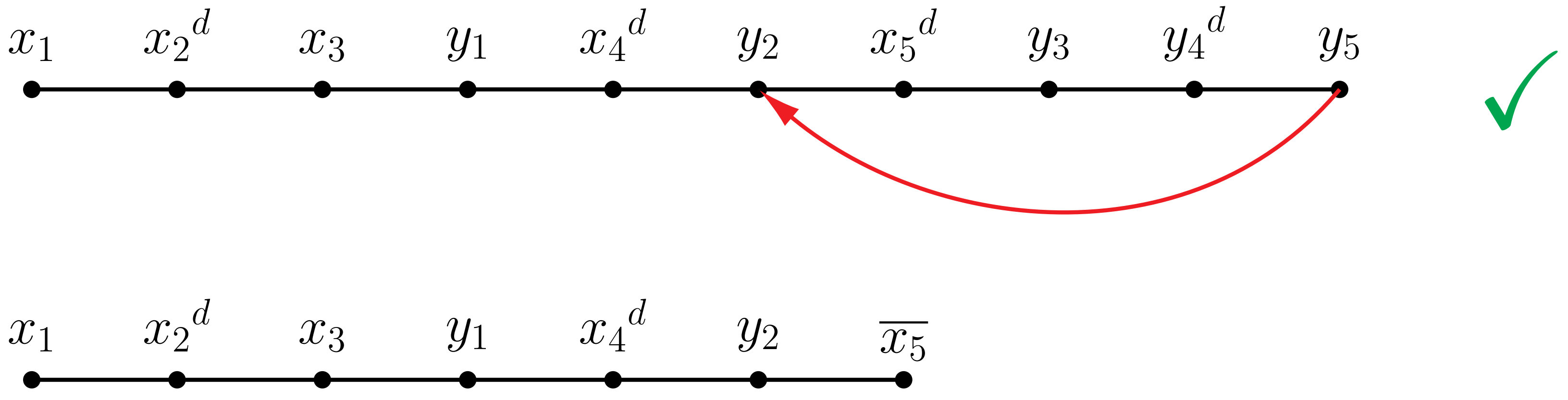

BackTrue: $(F,\ I,\ M,\ \delta)$

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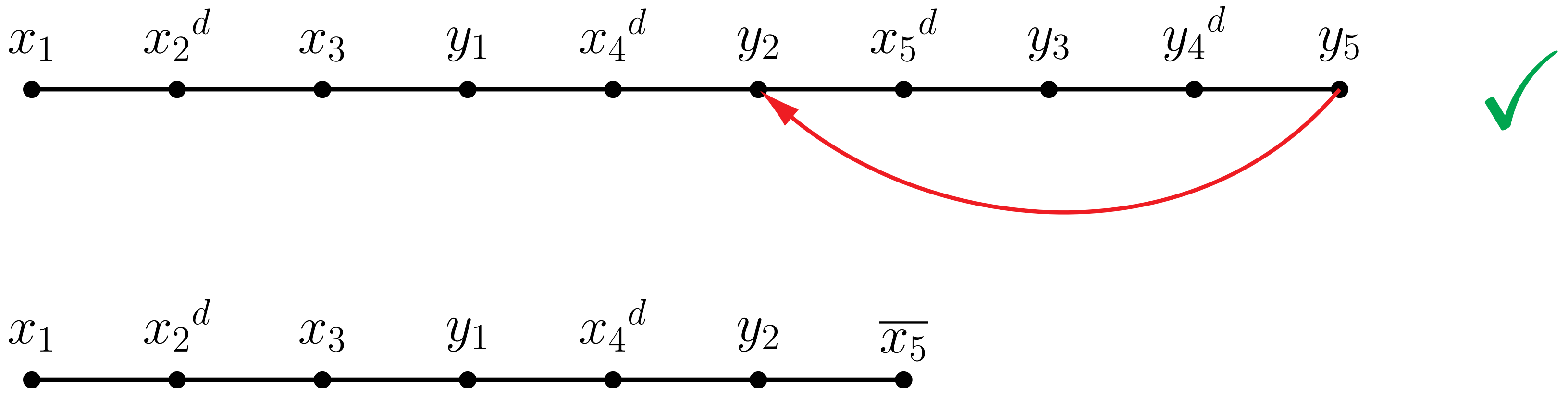

BackTrue: $(F,\ I,\ M,\ \delta)$ if $\forall X \exists Y\, [\, F|_I \,] = 1$

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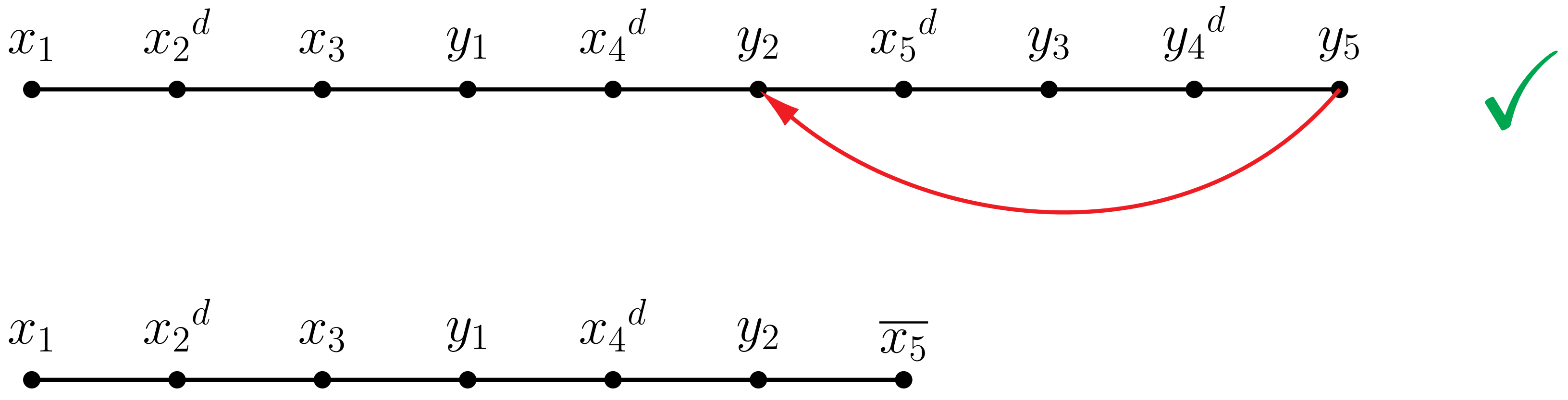

BackTrue: $(F,\ I,\ M,\ \delta)$ $\leadsto_{\text{BackTrue}}$ $(F,$ $)$ if $\forall X \exists Y\, [\, F|_I\,] = 1$

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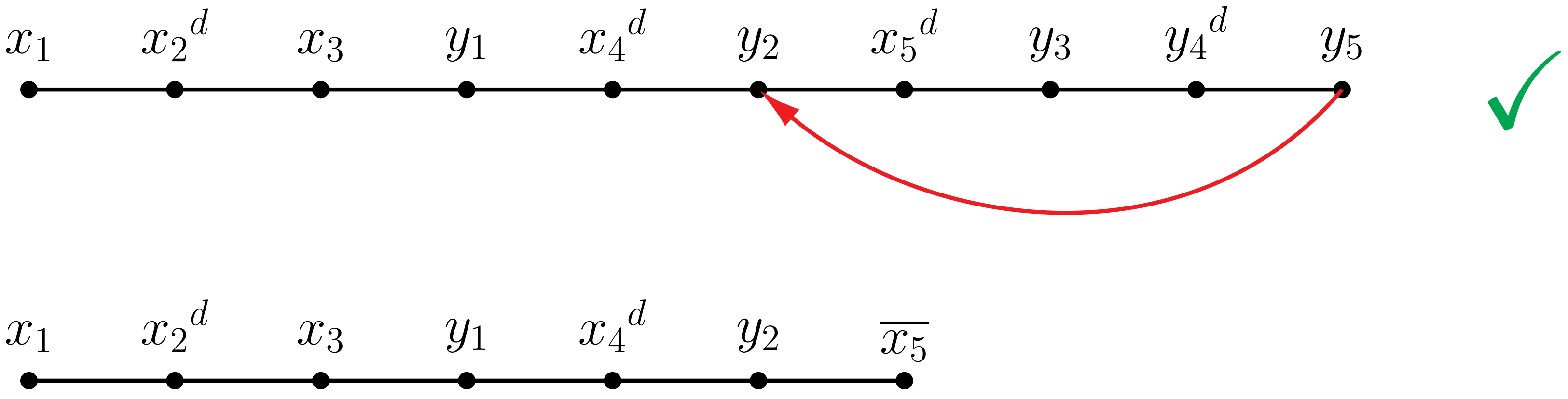

BackTrue: $(F,\ I,\ M,\ \delta) \rightsquigarrow_{\text{BackTrue}} (F,\quad M+m,\quad)$ if $\forall X \exists Y\, [\, F|_I\,] = 1$ and $m \stackrel{\text{def}}{=} 2^{|X-I|}$ and

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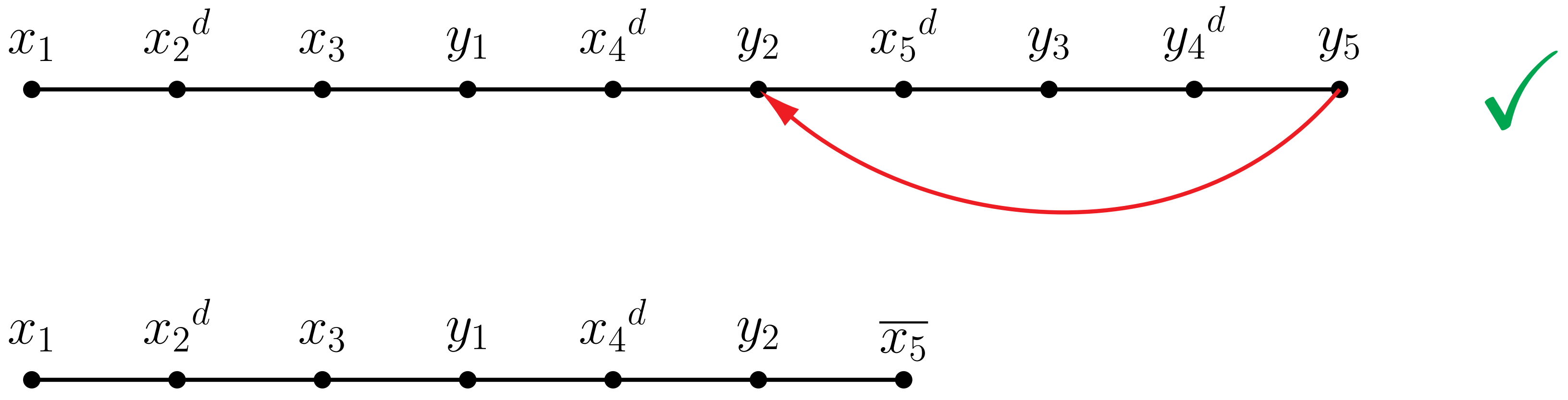

BackTrue: $(F,\ I,\ M,\ \delta) \leadsto_{\text{BackTrue}} (F,\ UK\ell,\ M+m,\qquad\qquad)$ if $\forall X \exists Y\, [\, F|_I\,] = 1$ and $m \overset{\text{def}}{=} 2^{|X-I|}$ and $D \overset{\text{def}}{=} \overline{\pi(\text{decs}(I), X)}$ and $\ell \in D$ and $UV \overset{\text{def}}{=} I$ and $K \overset{\text{def}}{=} V_{\leqslant b}$ and

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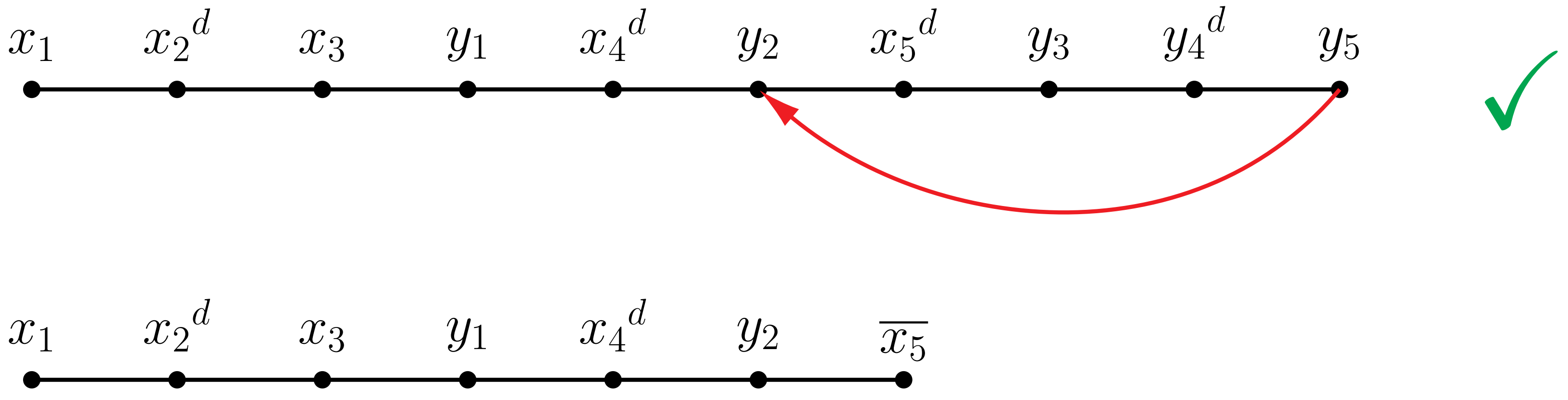

BackTrue: $(F,\ I,\ M,\ \delta) \leadsto_{\text{BackTrue}} (F,\ UK\ell,\ M+m,\ \delta[L \mapsto \infty][\ell \mapsto b])$ if $\forall X \exists Y\, [\, F|_I \,] = 1$ and $m \stackrel{\text{def}}{=} 2^{|X-I|}$ and $D \stackrel{\text{def}}{=} \overline{\pi(\text{decs}(I), X)}$ and $\ell \in D$ and $UV \stackrel{\text{def}}{=} I$ and $K \stackrel{\text{def}}{=} V_{\leqslant b}$ and $b = \delta(D \setminus \{\ell\}) = \delta(U)$ and $b + 1 \stackrel{\text{def}}{=} \delta(D) \leqslant \delta(I)$ and $L \stackrel{\text{def}}{=} V_{>b}$

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Given: Formula $F(X, Y)$ over relevant variables X and irrelevant variables Y

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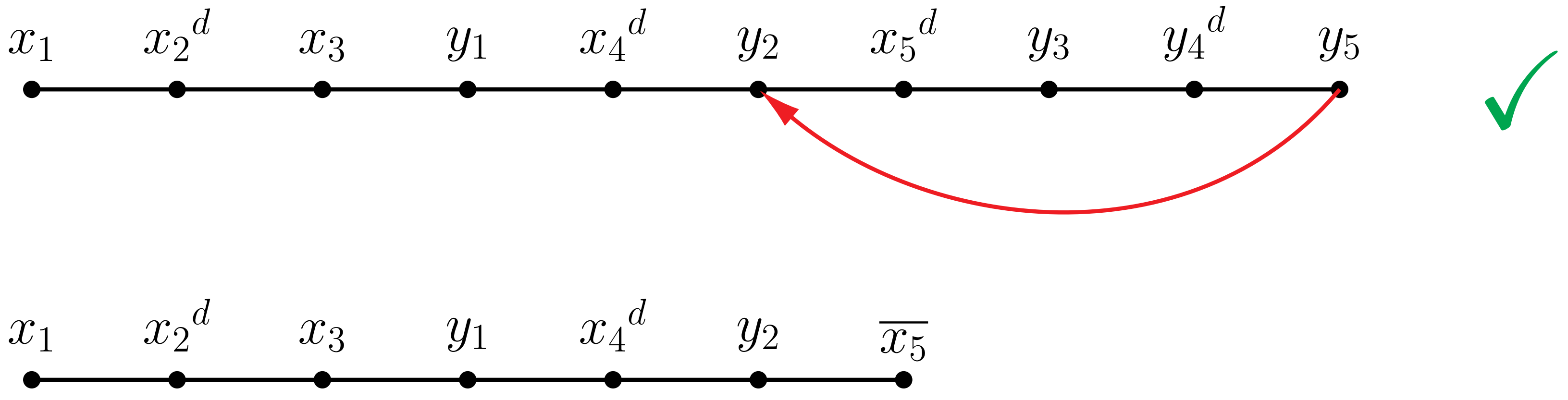

BackTrue: $(F,\, I,\, M,\, \delta) \leadsto_{\mathsf{BackTrue}} (F,\, UK\ell,\, M + m,\, \delta[L \mapsto \infty][\ell \mapsto b])$ if $\forall X \exists Y\, [\, F|_I\,] = 1$ and $m \stackrel{\text{def}}{=} 2^{|X - I|}$ and $D \stackrel{\text{def}}{=} \overline{\pi(\mathsf{decs}(I), X)}$ and $\ell \in D$ and $UV \stackrel{\text{def}}{=} I$ and $K \stackrel{\text{def}}{=} V_{\leqslant b}$ and $b = \delta(D \setminus \{\ell\}) = \delta(U)$ and $b + 1 \stackrel{\text{def}}{=} \delta(D) \leqslant \delta(I)$ and $L \stackrel{\text{def}}{=} V_{>b}$

Calculus

EndTrue: $(F, I, M, \delta) \rightsquigarrow_{\text{EndTrue}} M + m$ if $V(\text{decs}(I)) \cap X = \emptyset$ and $m \stackrel{\text{def}}{=} 2^{|X-I|}$ and $\forall X \exists Y\, [\, F|_I \,] = 1$

EndFalse: $(F, I, M, \delta) \rightsquigarrow_{\text{EndFalse}} M$ if exists $C \in F$ and $C|_I = 0$ and $\delta(C) = 0$

Unit: $(F, I, M, \delta) \rightsquigarrow_{\text{Unit}} (F, I\ell, M, \delta[\ell \mapsto a])$ if $F|_I \neq 0$ and exists $C \in F$ with $\{\ell\} = C|_I$ and $a \stackrel{\text{def}}{=} \delta(C \setminus \{\ell\})$

BackTrue: $(F, I, M, \delta) \rightsquigarrow_{\text{BackTrue}} (F, UK\ell, M + m, \delta[L \mapsto \infty][\ell \mapsto b])$ if $UV \stackrel{\text{def}}{=} I$ and $D \stackrel{\text{def}}{=} \overline{\pi(\text{decs}(I), X)}$ and $b + 1 \stackrel{\text{def}}{=} \delta(D) \leqslant \delta(I)$ and $\ell \in D$ and $b = \delta(D \setminus \{\ell\}) = \delta(U)$ and $m \stackrel{\text{def}}{=} 2^{|X-I|}$ and $K \stackrel{\text{def}}{=} V_{\leqslant b}$ and $L \stackrel{\text{def}}{=} V_{>b}$ and $\forall X \exists Y\, [\, F|_I \,] = 1$

BackFalse: $(F, I, M, \delta) \rightsquigarrow_{\text{BackFalse}} (F, UK\ell, M, \delta[L \mapsto \infty][\ell \mapsto j])$ if exists $C \in F$ and exists D with $UV \stackrel{\text{def}}{=} I$ and $C|_I = 0$ and $c \stackrel{\text{def}}{=} \delta(C) = \delta(D) > 0$ such that $\ell \in D$ and $\bar{\ell} \in \text{decs}(I)$ and $\bar{\ell}|_V = 0$ and $F \wedge \overline{M} \models D$ and $j \stackrel{\text{def}}{=} \delta(D \setminus \{\ell\})$ and $b \stackrel{\text{def}}{=} \delta(U) = c - 1$ and $K \stackrel{\text{def}}{=} V_{\leqslant b}$ and $L \stackrel{\text{def}}{=} V_{>b}$

DecideX: $(F, I, M, \delta) \rightsquigarrow_{\text{DecideX}} (F, I\ell^d, M, \delta[\ell \mapsto d])$ if $F|_I \neq 0$ and $\text{units}(F|_I) = \emptyset$ and $\delta(\ell) = \infty$ and $d \stackrel{\text{def}}{=} \delta(I) + 1$ and $V(\ell) \in X$

DecideY: $(F, I, M, \delta) \rightsquigarrow_{\text{DecideY}} (F, I\ell^d, M, \delta[\ell \mapsto d])$ if $F|_I \neq 0$ and $\text{units}(F|_I) = \emptyset$ and $\delta(\ell) = \infty$ and $d \stackrel{\text{def}}{=} \delta(I) + 1$ and $V(\ell) \in Y$ and $X - I = \emptyset$

Conclusion

Our Contribution

Method for computing the model count of a formula exploiting logical entailment

- Inspired by the interaction of theory and SAT solvers in SMT
- Adaptation of our method addressing projected partial model enumeration presented at SAT'20

Entailment test in four flavors of increasing strength (in the SAT'20 paper)

- $F|_I = 1$ (syntactic check)
- $F|_I \approx 1$ (incomplete check in $\mathbf{P}$)
- $F|_I \equiv 1$ (semantic check in $\mathbf{coNP}$)
- $\forall X \exists Y \, [F|_I] = 1$ (check in $\mathbf{\Pi}_2^P$)

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Further Research

- Implement and validate our method on instances stemming from weighted model integration and model counting with or without projection
- Investigate methods concerning the implementation of QBF oracles (Incremental QBF (Lonsing and Egly, CP'14))
- Combine with decomposition-based approaches (to generate a d-DNNF)